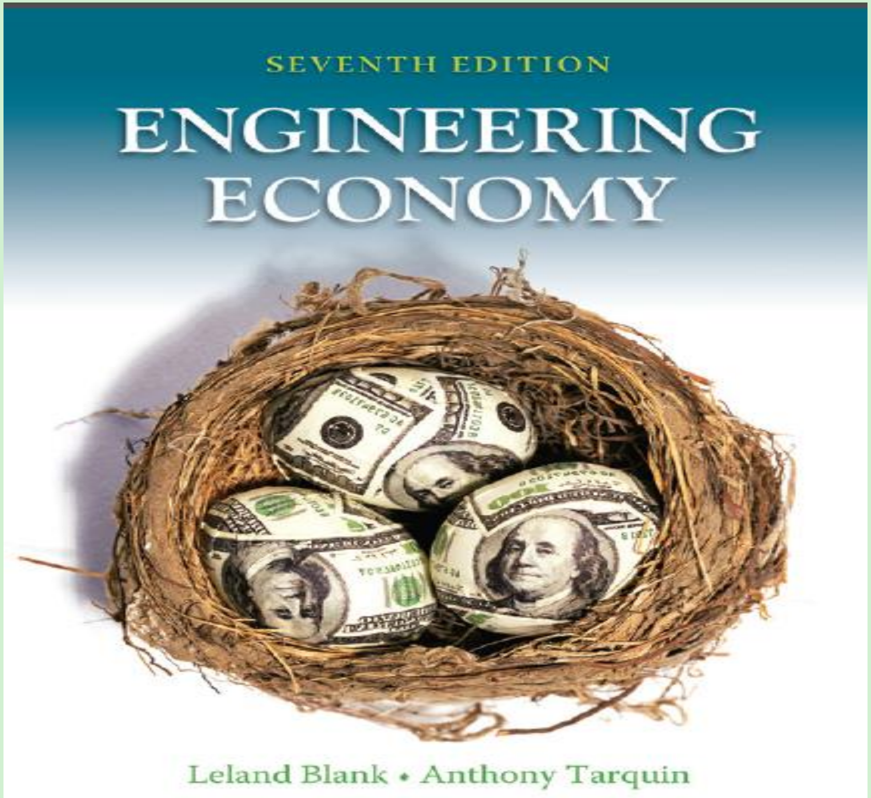
صندوق پژوهش و فناوری
خراسان رضوی



First session 1

Foundations Of Engineering Economy

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Course Website: http://www. Krrtf.ir & http:// newdeal.blogfa.com

Course Text: Engineering Economy, 7th edition,by Leland Blank & Anthony Tarquin

LEARNING OUTCOMES

Time Value of Money (TVM)

Interest rate

Terms and symbols

Cash flows

Economic equivalence

Simple and compound interest

Minimum attractive rate of return

Spreadsheet functions

Time Value of Money (TVM)

Description: TVM explains the change in the amount of money over time for funds owed by or owned by a corporation (or individual)

- Corporate investments are expected to earn a return
- Investment involves money
- Money has a 'time value'

The time value of money is the most important concept in engineering economy

Interest and Interest Rate

- **Interest** – the manifestation of the time value of money
  - Fee that one pays to use someone else's money
  - Difference between an ending amount of money and a beginning amount of money

  - **Interest = amount owed now – principal**

- **Interest rate** – Interest paid over a time period expressed as a percentage of principal

  - $$\text{Interest rate (\%)} = \frac{\text{interest accrued per time unit}}{\text{principal}} \times 100\%$$

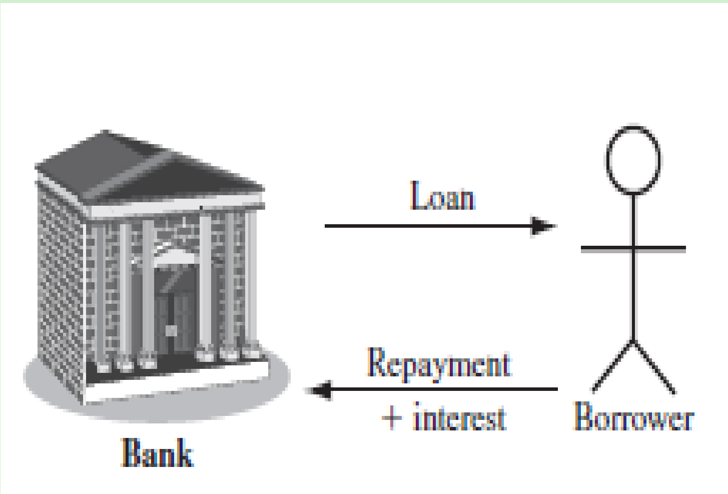
Rate of Return

- **Interest earned over a period of time is expressed as a percentage of the original amount (principal)**

$$\text{Rate of return (\%)} = \frac{\text{interest accrued per time unit}}{\text{original amount}} \times 100\%$$

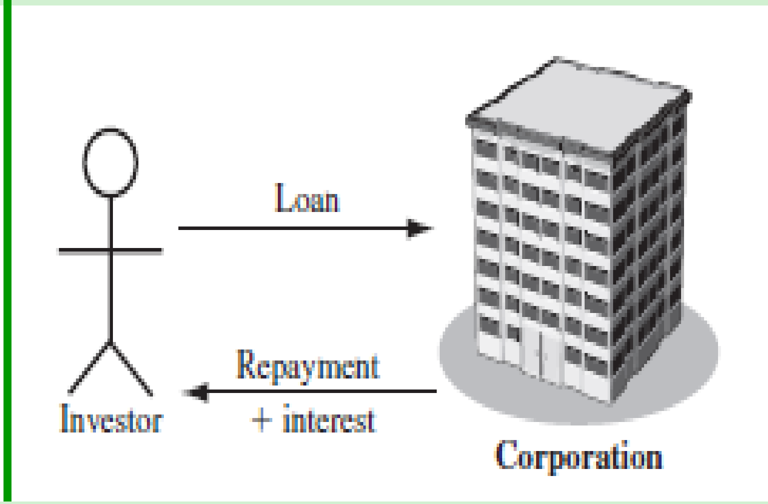
- Borrower's perspective – interest rate paid
- Lender's or investor's perspective – rate of return earned

Interest paid



Interest rate

Interest earned



Rate of return

Commonly used Symbols

t = time, usually in periods such as years or months

P = value or amount of money at a time *t* designated as present or time 0

F = value or amount of money at some future time, such as at *t* = *n* periods in the future

A = series of consecutive, equal, end-of-period amounts of money

n = number of interest periods; years, months

i = interest rate or rate of return per time period; percent per year or month

Cash Flows: Terms

- Cash Inflows – Revenues (R), receipts, incomes, savings generated by projects and activities that flow in. Plus sign used
- Cash Outflows – Disbursements (D), costs, expenses, taxes caused by projects and activities that flow out. Minus sign used
- Net Cash Flow (NCF) for each time period:

  NCF = cash inflows – cash outflows = R – D

- End-of-period assumption:

  Funds flow at the end of a given interest period

Cash Flows: Estimating

- ✓ **Point estimate – A single-value estimate of a cash flow element of an alternative**

  Cash inflow: Income = $150,000 per month

- ✓ **Range estimate – Min and max values that estimate the cash flow**

  Cash outflow: Cost is between $2.5 M and $3.2 M

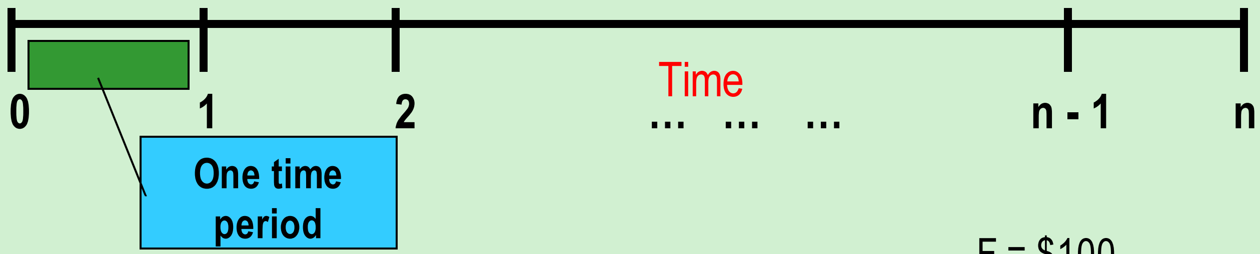
> Point estimates are commonly used; however, range estimates with probabilities attached provide a better understanding of variability of economic parameters used to make decisions

Cash Flow Diagrams

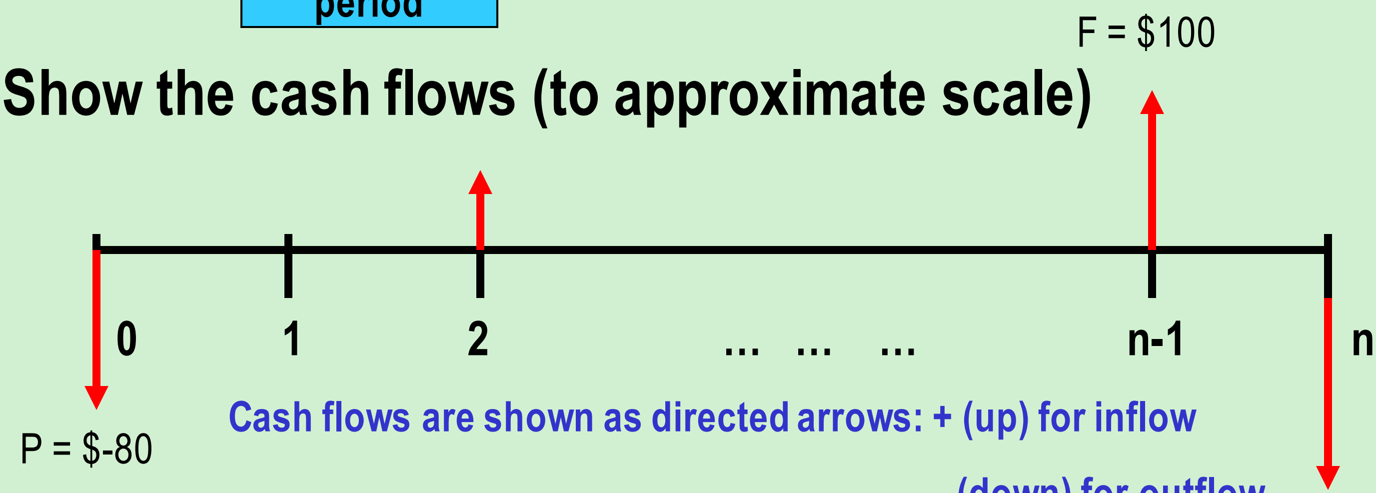
What a typical cash flow diagram might look like

Draw a time line

Always assume end-of-period cash flows



Show the cash flows (to approximate scale)



Cash flows are shown as directed arrows: + (up) for inflow

- (down) for outflow

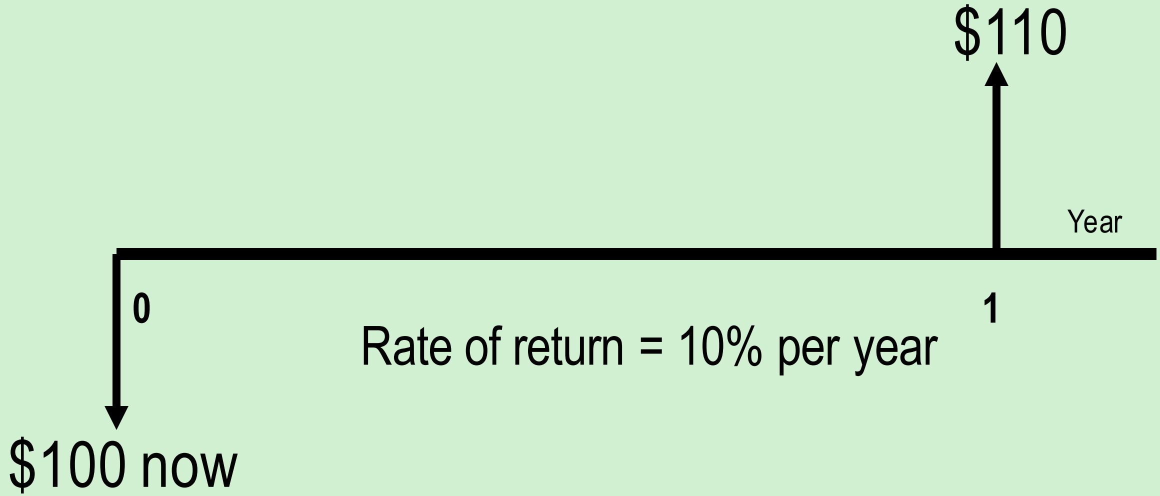
Economic Equivalence

Definition: Combination of interest rate (rate of return) and time value of money to determine different amounts of money at different points in time that are economically equivalent

How it works: Use rate *i* and time *t* in upcoming relations to move money (values of P, F and A) between time points *t* = 0, 1, …, n to make them equivalent (not equal) at the rate *i*

Example of Equivalence

Different sums of money at different times may be equal in economic value at a given rate



$100 now is economically equivalent to $110 one year from now, if the $100 is invested at a rate of 10% per year.

Simple and Compound Interest

- Simple Interest

Interest is calculated using principal only

Interest = (principal)(number of periods)(interest rate)

$I = Pni$

Example: $100,000 lent for 3 years at simple i = 10% per year. What is repayment after 3 years?

Interest = 100,000(3)(0.10) = $30,000

Total due = 100,000 + 30,000 = $130,000

Simple and Compound Interest

- Compound Interest

Interest is based on principal plus all accrued interest

That is, interest compounds over time

Interest = (principal + all accrued interest) (interest rate)

Interest for time period *t* is

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_j\right)(i)$$

Compound Interest Example

Example: $100,000 lent for 3 years at *i* = 10% per year compounded. What is repayment after 3 years?

Interest, year 1: $I_1 = 100{,}000(0.10) = \$10{,}000$

Total due, year 1: $T_1 = 100{,}000 + 10{,}000 = \$110{,}000$

Interest, year 2: $I_2 = 110{,}000(0.10) = \$11{,}000$

Total due, year 2: $T_2 = 110{,}000 + 11{,}000 = \$121{,}000$

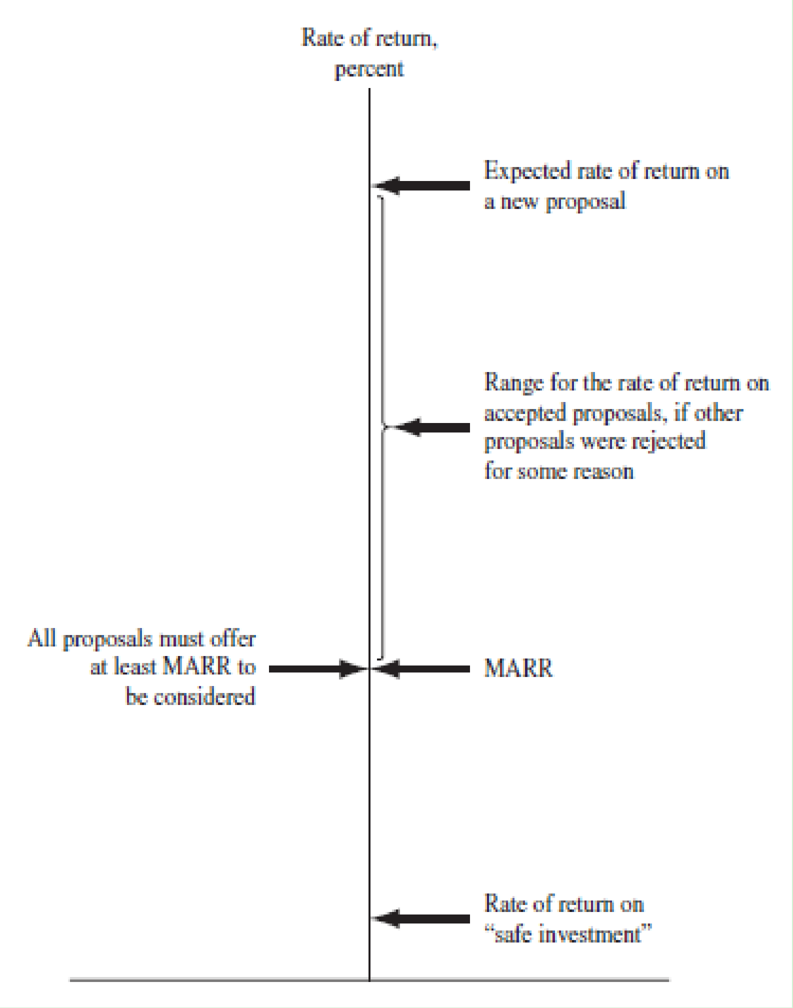
Interest, year 3: $I_3 = 121{,}000(0.10) = \$12{,}100$

Total due, year 3: $T_3 = 121{,}000 + 12{,}100 =$ **$133,100**

Compounded: $133,100	Simple: $130,000

Minimum Attractive Rate of Return

- MARR is a reasonable rate of return (percent) established for evaluating and selecting alternatives
- An investment is justified economically if it is expected to return at least the MARR
- Also termed *hurdle rate, benchmark rate* and *cutoff rate*



MARR Characteristics

- MARR is established by the financial managers of the firm
- MARR is fundamentally connected to the cost of capital
- Both types of capital financing are used to determine the weighted average cost of capital (WACC) and the MARR
- MARR usually considers the risk inherent to a project

Types of Financing

- **Equity Financing** –Funds either from retained earnings, new stock issues, or owner's infusion of money.
- **Debt Financing** –Borrowed funds from outside sources – loans, bonds, mortgages, venture capital pools, etc. Interest is paid to the lender on these funds

For an economically justified project

$$ROR \geq MARR > WACC$$

Opportunity Cost

- Definition: Largest rate of return of all projects not accepted (forgone) due to a lack of capital funds
- If no MARR is set, the ROR of the first project not undertaken establishes the opportunity cost

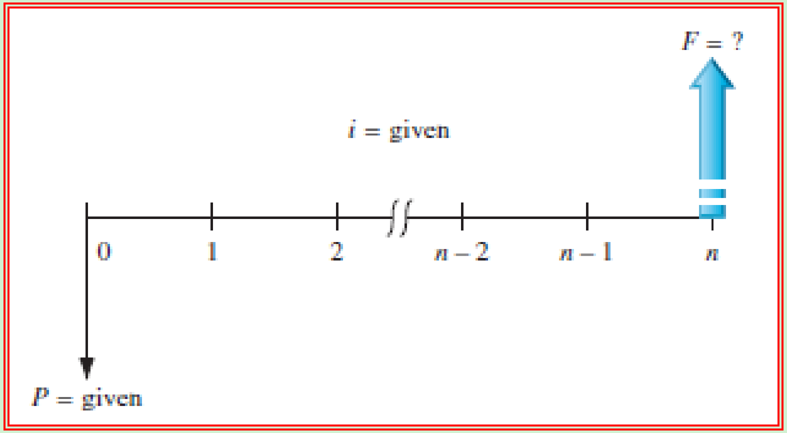
Example: Assume MARR = 10%. Project A, not funded due to lack of funds, is projected to have ROR_A = 13%. Project B has ROR_B = 15% and is funded because it costs less than A

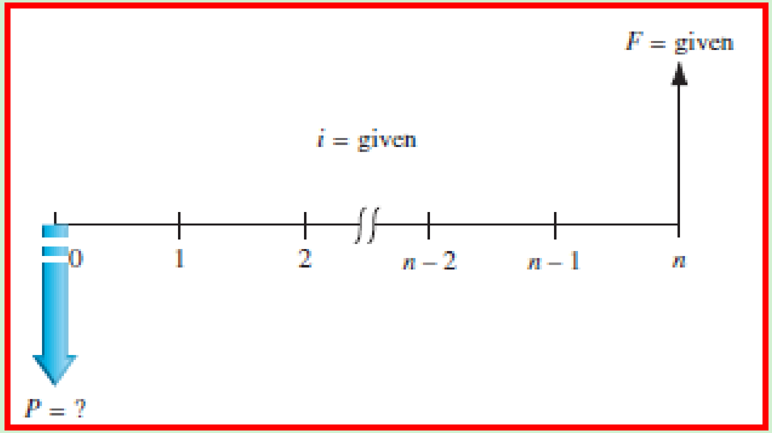
Opportunity cost is 13%, i.e., the opportunity to make an additional 13% is forgone by not funding project A

Single Payment Factors (F/P and P/F)

Single payment factors involve only P and F.

Cash flow diagrams are as follows:





Formulas are as follows:

$F = P(1 + i)^n$

$P = F[1 / (1 + i)^n]$

Terms in parentheses or brackets are called *factors.* Values are in tables for i and n values

Factors are represented in *standard factor notation* *such as (F/P,i,n)*,

where letter to left of slash is what is sought; letter to right represents what is given

F/P and P/F for Spreadsheets

Future value F is calculated using FV function:

= FV(i%,n,,P)

Present value P is calculated using PV function:

= PV(i%,n,,F)

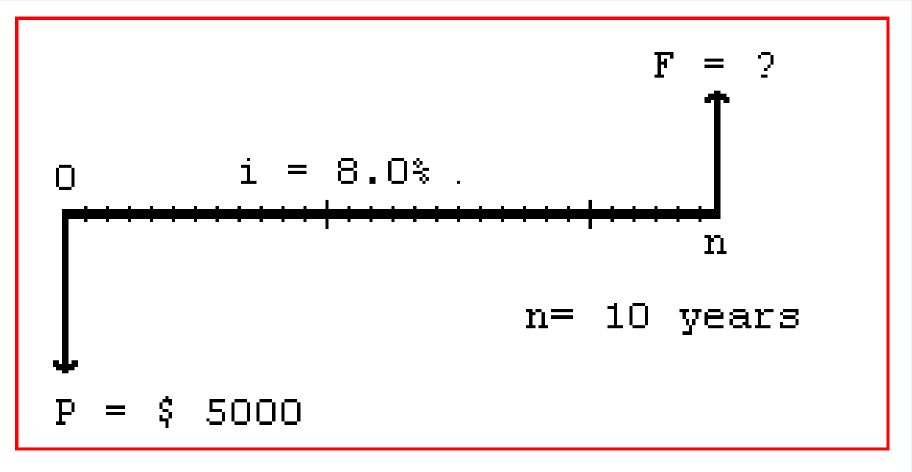
Note the use of double commas in each function

Example: Finding Future Value

A person deposits $5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

(A) $2,792 (B) $9,000 (C) $10,795 (D) $12,165

The cash flow diagram is:



Solution:

F = P(F/P,i,n)

= 5000(F/P,8%,10)

= 5000(2.1589)

= $10,794.50

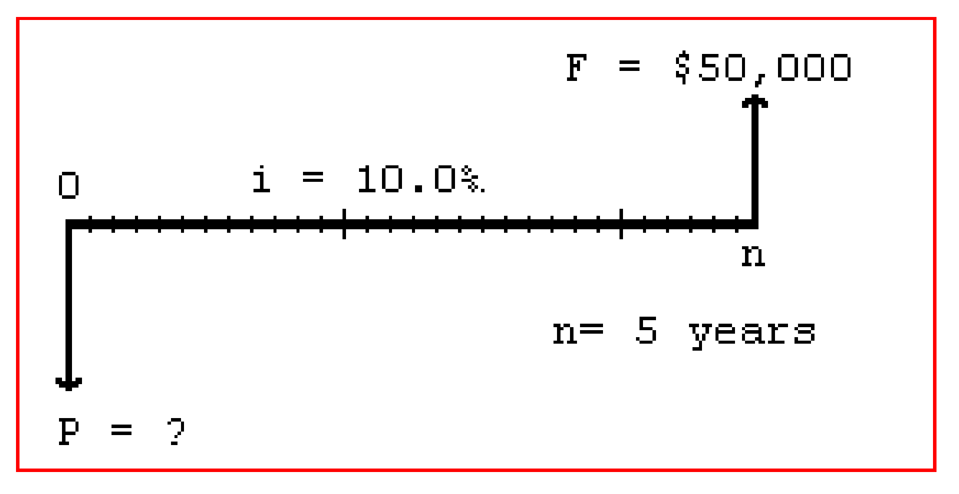
Answer is (C)

Example: Finding Present Value

A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

(A) $10,000 (B) $ 31,050 (C) $ 33,250 (D) $319,160

The cash flow diagram is:



Solution:

P = F(P/F,i,n)

= 50,000(P/F,10%,5)

= 50,000(0.6209)

= $31,045

Answer is (B)

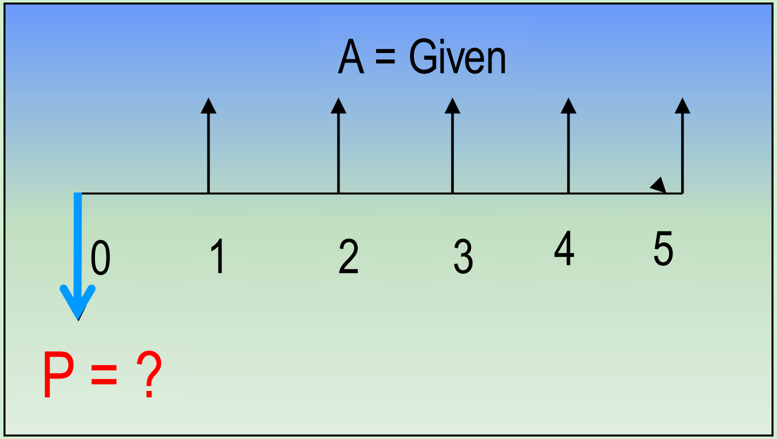
Uniform Series Involving P/A and A/P

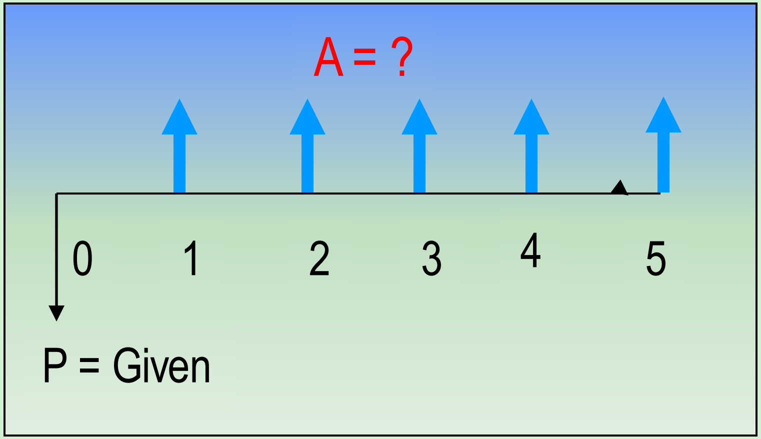
The uniform series factors that involve **P and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Cash flow amount is ***same*** in each interest period

The cash flow diagrams are:





P = A(P/A,i,n) ⟵ Standard Factor Notation ⟶ A = P(A/P,i,n)

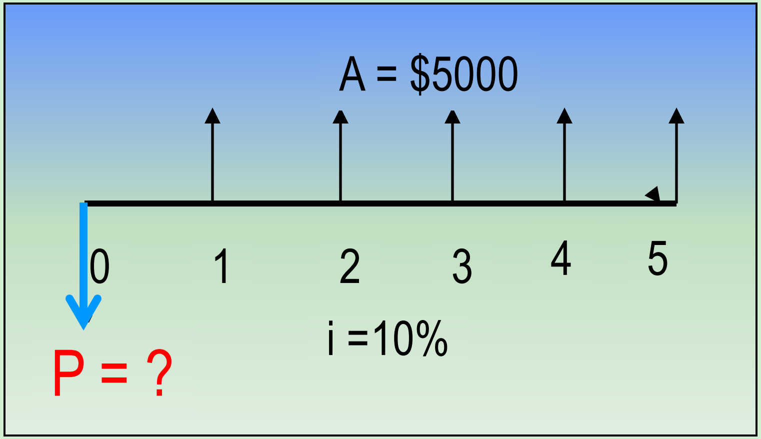
Note: P is one period *Ahead* of first A value

Example: Uniform Series Involving P/A

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra $5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

(A) $11,170 **(B) 13,640** **(C) $15,300** **(D) $18,950**

The cash flow diagram is as follows:



Solution:

$P = 5000(P/A,10\%,5)$

$= 5000(3.7908)$

$= \$18,954$

Answer is (D)

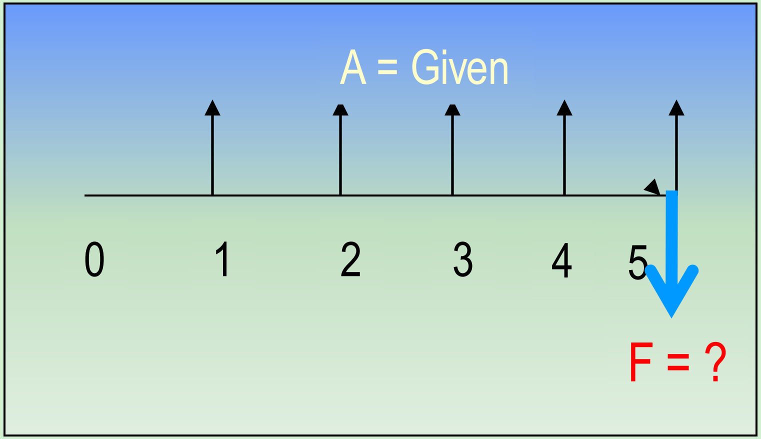
Uniform Series Involving F/A and A/F

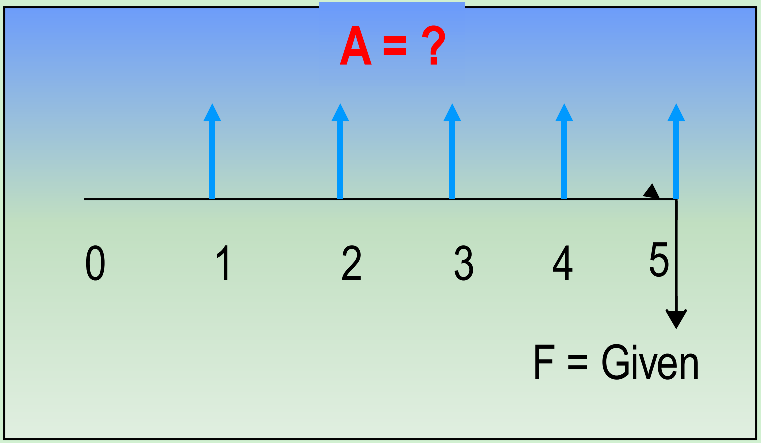
The uniform series factors that involve **F and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Last cash flow occurs in ***same*** period as F

Cash flow diagrams are:





F = A(F/A,i,n) ⟵ Standard Factor Notation ⟶ A = F(A/F,i,n)

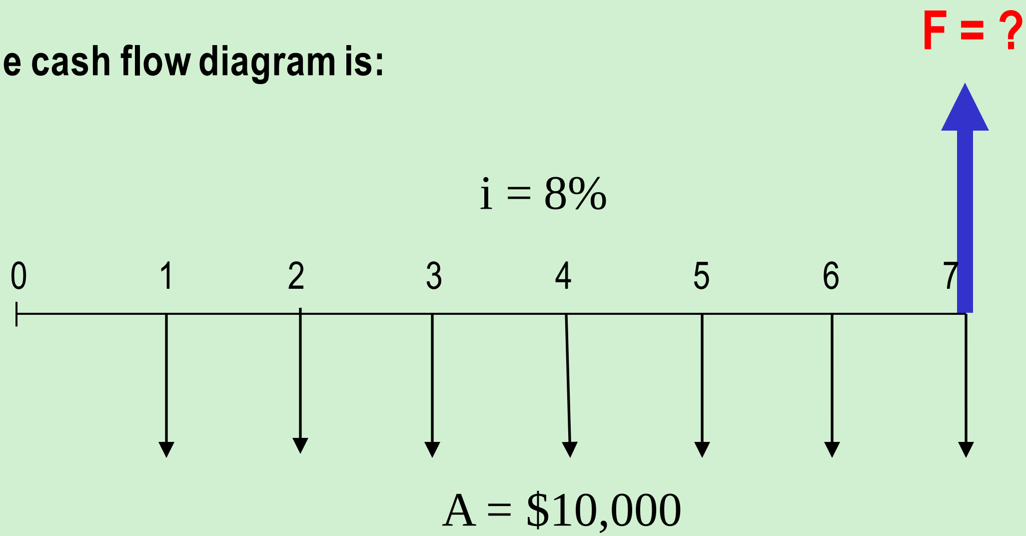
Note: F takes place in the *same* period as last A

Example: Uniform Series Involving F/A

An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

(A) $45,300 (B) $68,500 (C) $89,228 (D) $151,500

The cash flow diagram is:



Solution:

$$F = 10{,}000(F/A,8\%,7)$$

$$= 10{,}000(8.9228)$$

$$= \$89{,}228$$

Answer is (C)

Factor Values for Untabulated i or n

3 ways to find factor values for untabulated i or n values

- **Use formula**
- **Use spreadsheet function with corresponding P, F, or A value set to 1**
- **Linearly interpolate in interest tables**

Formula or spreadsheet function is fast and accurate
Interpolation is only approximate

Example: Untabulated i

Determine the value for (F/P, 8.3%,10)

Formula: $F = (1 + 0.083)^{10} = 2.2197$ ⟸ **OK**

Spreadsheet: = FV(8.3%,10,,1) = 2.2197 ⟸ **OK**

Interpolation: 8% ------ 2.1589
8.3% ------ x
9% ------ 2.3674

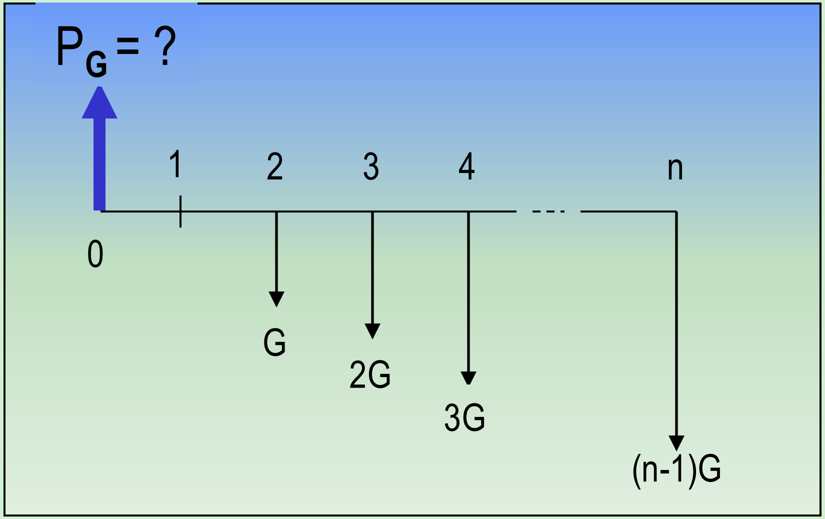
x = 2.1589 + [(8.3 - 8.0)/(9.0 - 8.0)][2.3674 – 2.1589]
= 2.2215 ⟸ **(Too high)**

Absolute Error = 2.2215 – 2.2197 = 0.0018

Arithmetic Gradients

Arithmetic gradients *change* by the *same amount* each period

The cash flow diagram for the P_G of an arithmetic gradient is:



G starts between periods 1 and 2
(not between 0 and 1)

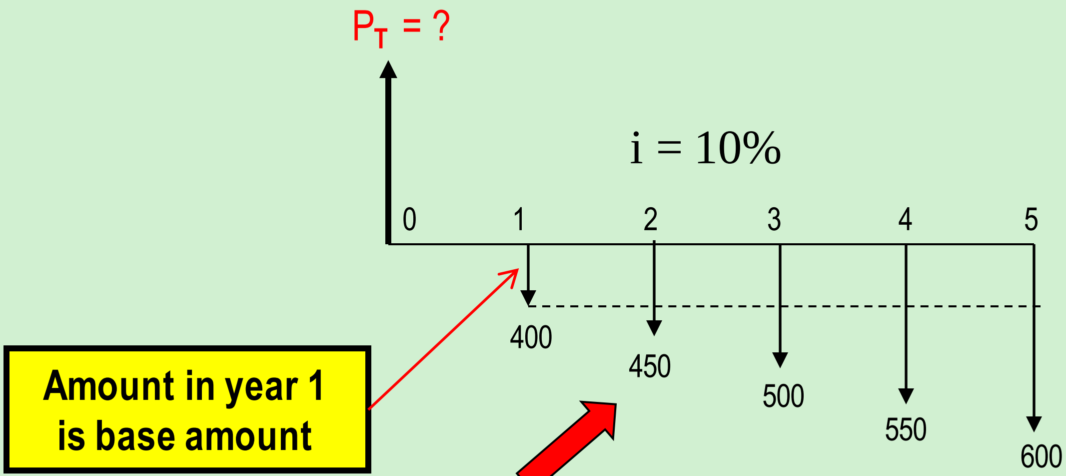
This is because cash flow in year 1 is usually not equal to G and is handled separately as a *base amount* (shown on next slide)

Note that P_G is located Two Periods Ahead of the first change that is equal to G

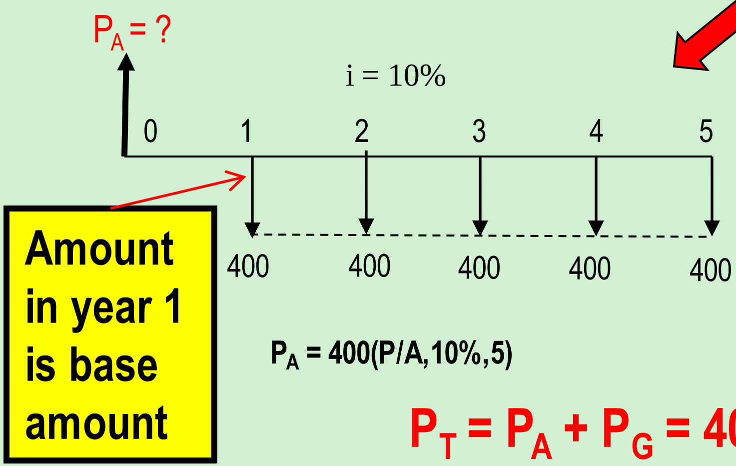
Standard factor notation is

$P_G = G(P/G,i,n)$

Typical Arithmetic Gradient Cash Flow

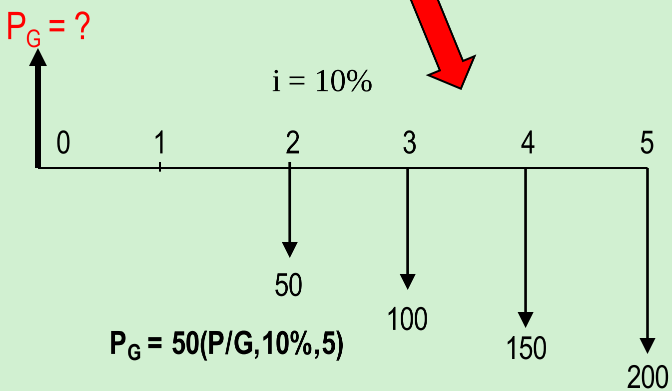


This diagram = this *base amount* plus this gradient



$P_A = 400(P/A,10\%,5)$

+

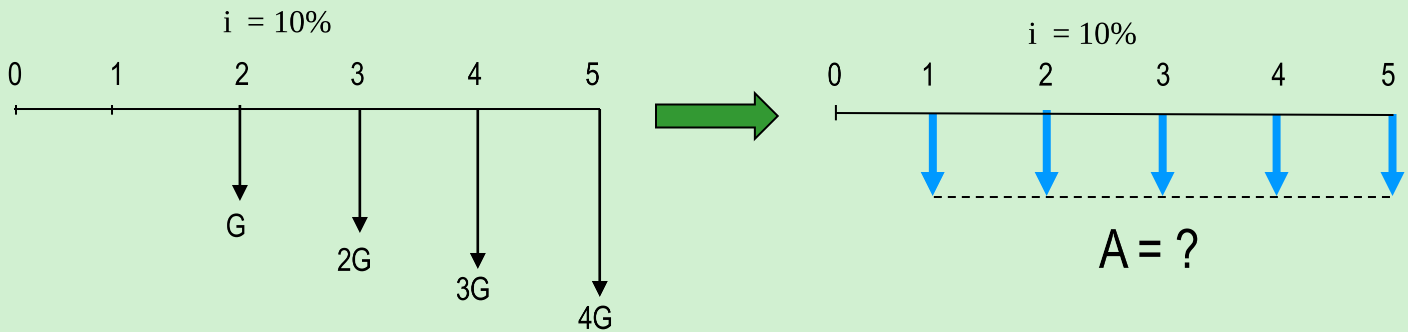


$P_G = 50(P/G,10\%,5)$

$$P_T = P_A + P_G = 400(P/A,10\%,5) + 50(P/G,10\%,5)$$

Converting Arithmetic Gradient to A

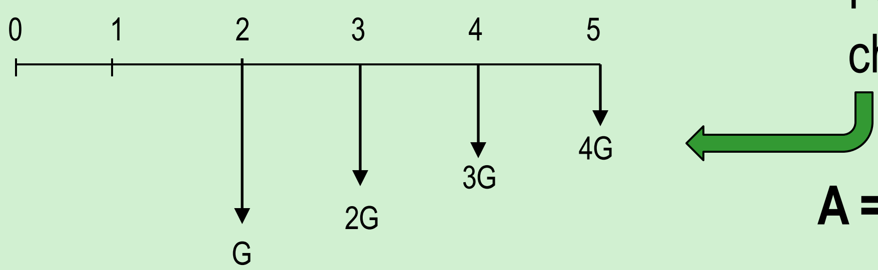
Arithmetic gradient can be converted into equivalent A value using G(A/G,i,n)



General equation when base amount is involved is

A = base amount + G(A/G,i,n)

For decreasing gradients, change plus sign to minus

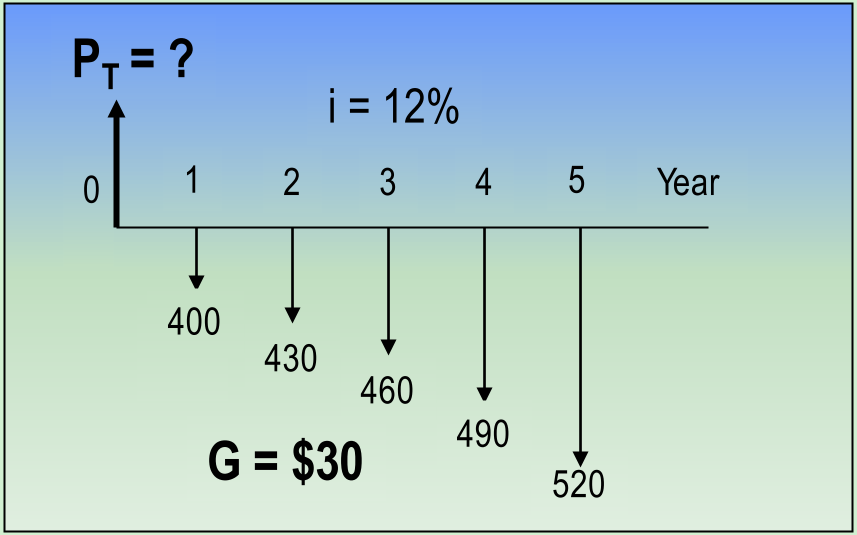


A = base amount - G(A/G,i,n)

Example: Arithmetic Gradient

The present worth of $400 in year 1 and amounts increasing by $30 per year through year 5 at an interest rate of 12% per year is closest to:

(A) $1532 (B) $1,634 (C) $1,744 (D) $1,829



Solution:

P_T = 400(P/A,12%,5) + 30(P/G,12%,5)
= 400(3.6048) + 30(6.3970)
= $1,633.83

Answer is (B)

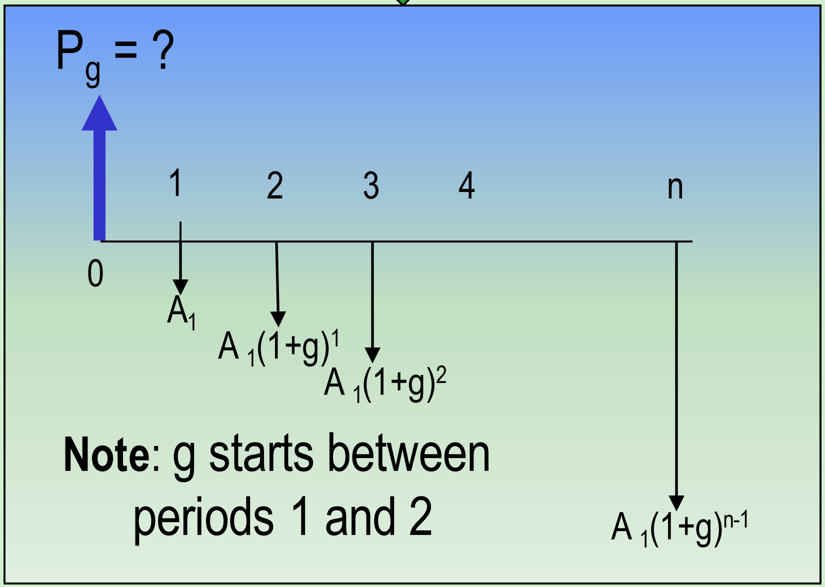
The cash flow could also be converted into an **A** value as follows:

A = 400 + 30(A/G,12%,5)
= 400 + 30(1.7746)
= $453.24

Geometric Gradients

Geometric gradients **change by the *same percentage* each period**

Cash flow diagram for present worth of geometric gradient



There are ***no tables*** for geometric factors

Use following equation for $g \neq i$:

$$P_g = A_1\{1 - [(1+g)/(1+i)]^n\}/(i-g)$$

where: A_1 *= cash flow in period 1*

g = rate of increase

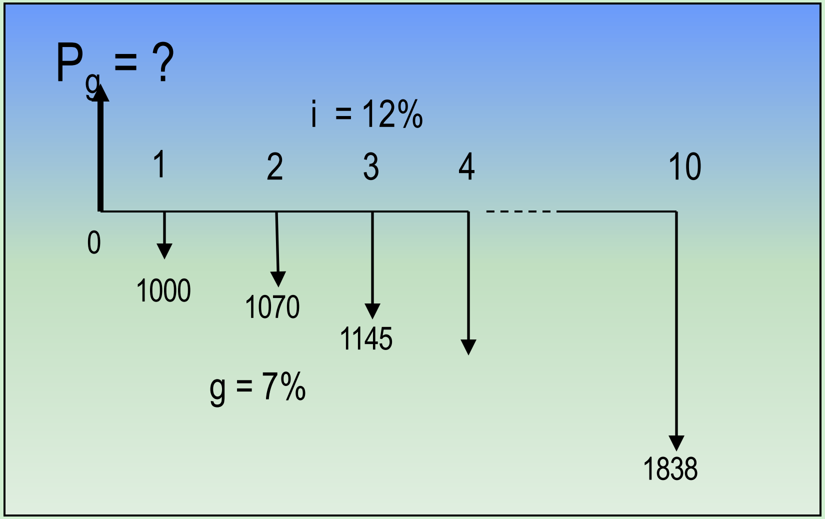
If $g = i$, $P_g = A_1 n/(1+i)$

Note: If g is negative, change signs in front of both g values

Example: Geometric Gradient

Find the present worth of $1,000 in year 1 and amounts increasing by 7% per year through year 10. Use an interest rate of 12% per year.

(a) $5,670 (b) $7,333 (c) $12,670 (d) $13,550



Solution:

$P_g = 1000[1-(1+0.07/1+0.12)^{10}]/(0.12-0.07)$

$= \$7,333$

Answer is (b)

To find A, multiply P_g by (A/P,12% ,10)

Unknown Interest Rate i

Unknown interest rate problems involve solving for i, given n and 2 other values (P, F, or A)

(Usually requires a trial and error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for i

A contractor purchased equipment for $60,000 which provided income of $16,000 per year for 10 years. The annual rate of return of the investment was closest to:

(a) 15% (b) 18% (c) 20% (d) 23%

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,i\%,10) = 16{,}000$$

$$(A/P,i\%,10) = 0.26667$$

From A/P column at n = 10 in the interest tables, i is between 22% and 24%

Answer is (d)

Unknown Recovery Period n

Unknown recovery period problems involve solving for n, given i and 2 other values (P, F, or A)

(Like interest rate problems, they usually require a trial & error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for n

A contractor purchased equipment for $60,000 that provided income of $8,000 per year. At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

(a) 10 years **(b) 12 years** **(c) 15 years** **(d) 18 years**

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,10\%,n) = 8{,}000$$

$$(A/P,10\%,n) = 0.13333$$

From A/P column in i = 10% interest tables, n is between 14 and 15 years **Answer is (c)**

Summary of Important Points

- In P/A and A/P factors, P is *one period ahead* of first A
- In F/A and A/F factors, F is in *same period as last* A
- To find untabulated factor values, best way is to use *formula or spreadsheet*
- For arithmetic gradients, gradient G starts between *periods 1 and 2*
- Arithmetic gradients have 2 parts, *base amount* (year 1) and *gradient amount*
- For geometric gradients, gradient g starts been *periods 1 and 2*
- In geometric gradient formula, A_1 is amount in *period 1*
- To find unknown i or n, *set up equation involving all terms* and solve for i or n